

2.5.3 Kvadratické funkce s absolutní hodnotou

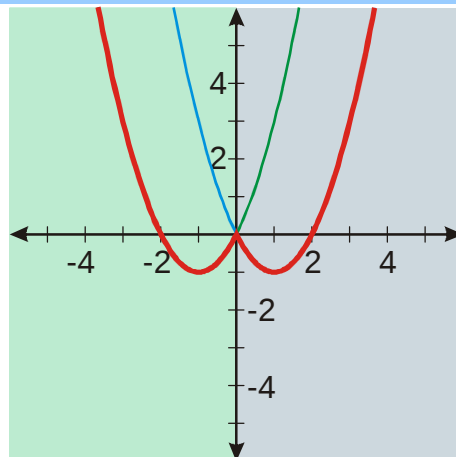
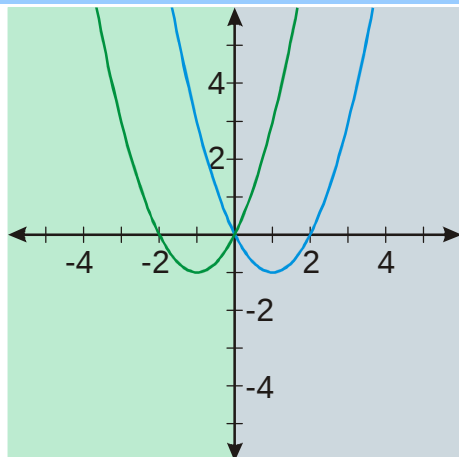
Př. 1: Nakresli graf funkce $y = x^2 - 2|x|$.

1) $x \in (-\infty; 0)$ $x < 0 \Rightarrow |x| = -x$

$$y = x^2 - 2|x| = x^2 - 2(-x) = x^2 + 2x + 1 - 1 = (x+1)^2 - 1$$

2) $x \in \langle 0; \infty)$ $x > 0 \Rightarrow |x| = x$

$$y = x^2 - 2|x| = x^2 - 2x = x^2 - 2x + 1 - 1 = (x-1)^2 - 1$$



Př. 2: Najdi způsob, jak s využitím vlastností funkce $y = x^2 - 2|x|$ nakreslit její graf bez nutnosti rozdělování definičního oboru na intervaly.

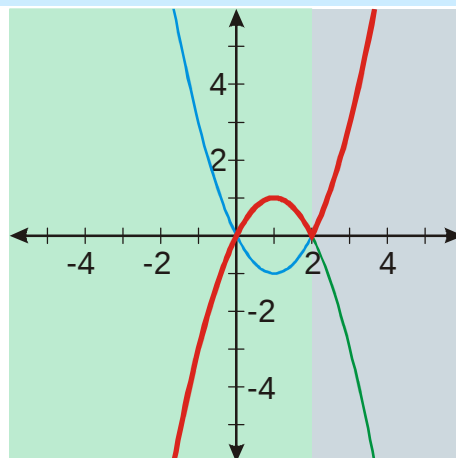
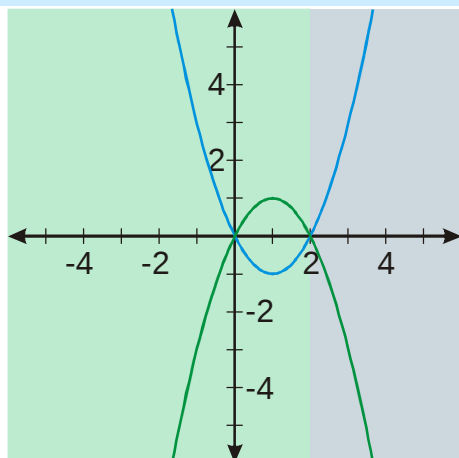
Př. 3: Nakresli graf funkce $y = x|x-2|$.

1) $x \in (-\infty; 2)$ $x - 2 < 0 \Rightarrow |x-2| = -x+2$

$$y = x|x-2| = x(-x+2) = -x^2 + 2x = -x^2 + 2x - 1 + 1 = -(x-1)^2 + 1$$

2) $x \in \langle 2; \infty)$ $x - 2 > 0 \Rightarrow |x-2| = x-2$

$$y = x|x-2| = x(x-2) = x^2 - 2x = x^2 - 2x + 1 - 1 = (x-1)^2 - 1$$



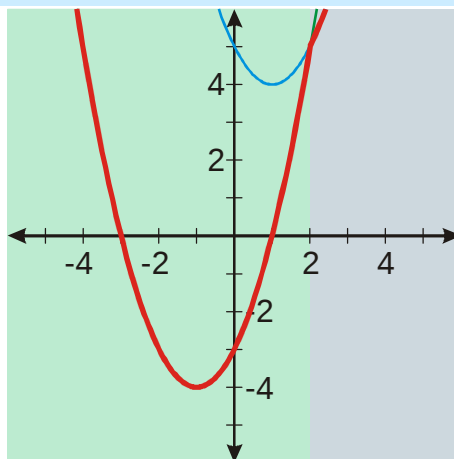
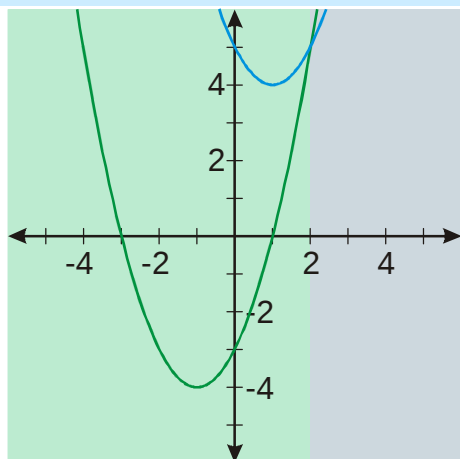
Př. 4: Nakresli graf funkce $y = x^2 - 2|x - 2| + 1$.

1) $x \in (-\infty; 2)$ $x - 2 < 0 \Rightarrow |x - 2| = -x + 2$

$$y = x^2 - 2|x - 2| + 1 = x^2 + 2(x - 2) + 1 = x^2 + 2x - 4 + 1 = (x^2 + 2x + 1) - 4 = (x + 1)^2 - 4$$

2) $x \in \langle 2; \infty)$ $x - 2 > 0 \Rightarrow |x - 2| = x - 2$

$$y = x^2 - 2|x - 2| + 1 = x^2 - 2(x - 2) + 1 = x^2 - 2x + 4 + 1 = (x^2 - 2x + 1) + 4 = (x - 1)^2 + 4$$



Př. 5: Nakresli graf funkce $y = x|2 - x| + 2|x| - 2$.

1) $x \in (-\infty; 0)$ $2 - x > 0 \Rightarrow |2 - x| = 2 - x$ $x < 0 \Rightarrow |x| = -x$

$$y = x|2 - x| + 2|x| - 2 = x(2 - x) + 2(-x) - 2 = 2x - x^2 - 2x - 2 = -x^2 - 2$$

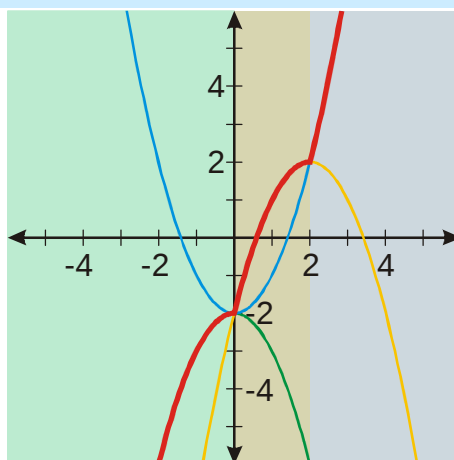
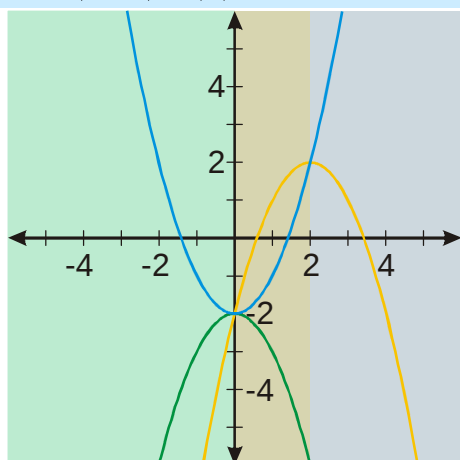
2) $x \in \langle 0; 2)$ $2 - x > 0 \Rightarrow |2 - x| = 2 - x$ $x > 0 \Rightarrow |x| = x$

$$y = x|2 - x| + 2|x| - 2 = x(2 - x) + 2x - 2 = 2x - x^2 + 2x - 2 = -x^2 + 4x - 2 =$$

$$= -(x^2 - 4x) - 2 = -(x^2 - 2x \cdot 2 + 2^2 - 2^2) - 2 = -[(x - 2)^2 - 4] - 2 = -(x - 2)^2 + 2$$

3) $x \in \langle 2; \infty)$ $2 - x < 0 \Rightarrow |2 - x| = -2 + x$ $x > 0 \Rightarrow |x| = x$

$$y = x|2 - x| + 2|x| - 2 = x(x - 2) + 2x - 2 = x^2 - 2x + 2x - 2 = x^2 - 2$$



Př. 6: Petáková:

strana 29/cvičení 55 $g_2; g_4; h_3; k_1$